1. $f(x) = 4 \operatorname{cosec} x - 4x + 1$, where x is in radians.

(a) Show that there is a root α of f (x) = 0 in the interval [1.2, 1.3].

(2)

(b) Show that the equation f(x) = 0 can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of f(x) in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)
(Total 9 marks)

2.

f(x) = 2x3 – x – 4.

(a) Show that the equation f(x) = 0 can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)
(Total 9 marks)

3. $f(x) = 3e^x - \frac{1}{2}\ln x - 2, \quad x > 0.$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \frac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

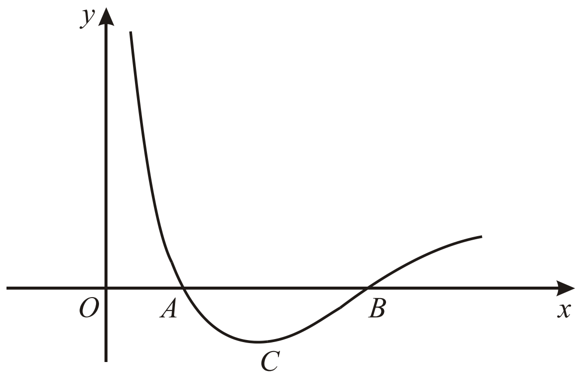
(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)
(Total 9 marks)

4.



$$f(x) = \frac{1}{2x} - 1 + \ln\frac{x}{2},\ x > 0.$$

The diagram above shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points A and B, and has a minimum at the point C.

(a) Show that the x-coordinate of C is $\frac{1}{2}$.

(5)

(b) Find the y-coordinate of C in the form $k \ln 2$, where k is a constant.

(2)

(c) Verify that the x-coordinate of B lies between 4.905 and 4.915.

(2)

(d) Show that the equation $\frac{1}{2x} - 1 + \ln\frac{x}{2} = 0$ can be rearranged into the form $x = 2e^{\left(1 - \frac{1}{2x}\right)}$.

(2)

The x-coordinate of B is to be found using the iterative formula

$$x_{n+1} = 2e^{\left(1 - \frac{1}{2x_n}\right)}, \qquad \text{with } x_0 = 5.$$

(e) Calculate, to 4 decimal places, the values of x_1, x_2 and x_3.

(2)
(Total 13 marks)

5.

$$f(x) = x^3 - 2 - \frac{1}{x}, \qquad x \neq 0.$$

(a) Show that the equation f(x) = 0 has a root between 1 and 2.

(2)

An approximation for this root is found using the iteration formula

$$x_{n+1} = \left(2 + \frac{1}{x_n}\right)^{\frac{1}{3}}, \qquad \text{with } x_0 = 1.5.$$

(b) By calculating the values of x_1, x_2, x_3 and x_4, find an approximation to this root, giving your answer to 3 decimal places.

(4)

(c) By considering the change of sign of f(x) in a suitable interval, verify that your answer to part (b) is correct to 3 decimal places.

(2)
(Total 8 marks)

6.

$$f(x) = x^3 + x^2 - 4x - 1.$$

The equation $f(x) = 0$ has only one positive root, α.

(a) Show that $f(x) = 0$ can be rearranged as

$$x = \sqrt{\left(\frac{4x+1}{x+1}\right)},\ x \neq -1.$$

(2)

The iterative formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 1$, find, to 2 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) By choosing values of x in a suitable interval, prove that $\alpha = 1.70$, *correct* to 2 decimal places.

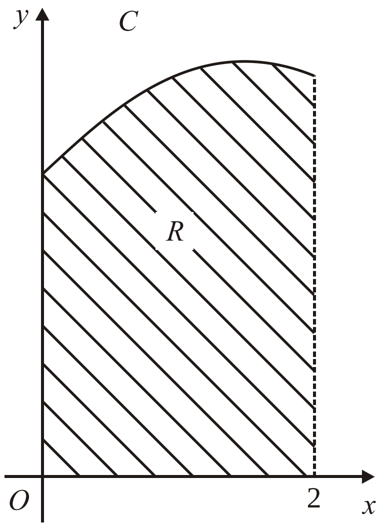
(3)

(d) Write down a value of x_1 for which the iteration formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ does *not* produce a valid value for x_2.

Justify your answer.

(2)
(Total 10 marks)

7.



The curve C has equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The diagram above shows the part of C for which $0 \leq x \leq 2$.

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x - 2x^2,$$

and that C has a single maximum, at $x = k$,

(a) show that $1.48 < k < 1.49$.

(3)

Given also that the point (0, 5) lies on C,

(b) find $\mathrm{f}(x)$.

(4)

The finite region R is bounded by C, the coordinate axes and the line $x = 2$.

(c) Use integration to find the exact area of R.

(4)
(Total 11 marks)

8. The curve C has equation $y = \text{f}(x)$, where

$$\text{f}(x) = 3\ln x + \frac{1}{x}, \qquad x > 0.$$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.

(4)

(b) Show that the y-coordinate of P may be expressed in the form $k - k\ln k$, where k is a constant to be found.

(2)

The point Q on C has x-coordinate 1.

(c) Find an equation for the normal to C at Q.

(4)

The normal to C at Q meets C again at the point R.

(d) Show that the x-coordinate of R

(i) satisfies the equation $6\ln x + x + \dfrac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14.

(4)
(Total 14 marks)

9. (a) Sketch, on the same set of axes, the graphs of

$$y = 2 - \text{e}^{-x} \quad \text{and } y = \sqrt{x}.$$

[It is not necessary to find the coordinates of any points of intersection with the axes.]

(3)

Given that $f(x) = e^{-x} + \sqrt{x} - 2, \quad x \geq 0,$

(b) explain how your graphs show that the equation $f(x) = 0$ has only one solution,

(1)

(c) show that the solution of $f(x) = 0$ lies between $x = 3$ and $x = 4$.

(2)

The iterative formula $x_{n+1} = (2 - e^{-x_n})^2$ is used to solve the equation $f(x) = 0$.

(d) Taking $x_0 = 4$, write down the values of x_1, x_2, x_3 and x_4, and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 3 decimal places.

(4)

(Total 10 marks)

1. (a) $f(1.2) = 0.49166551...,\quad f(1.3) = -0.048719817...$

Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$ M1 A1 2

Note

M1: Attempts to evaluate both f(1.2) and f(1.3) and evaluates at least one of them correctly to awrt (or truncated) 1 sf.

A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.

(b) $4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1$ M1

$\Rightarrow x = \operatorname{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$ A1 * 2

Note

M1: Attempt to make $4x$ or x the subject of the equation.
A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial f(x) = 0.

(c) $x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$ M1

$x_1 = 1.303757858...,\quad x_2 = 1.286745793...$ A1

$x_3 = 1.291744613...$ A1 3

Note

M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula

Eg = $\frac{1}{\sin(1.25)} + \frac{1}{4}$

Can be implied by x_1 = awrt 1.3 or x_1 = awrt 46°.

A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867

A1: x_3 = awrt 1.2917

(d) $f(1.2905) = 0.00044566695...,\quad f(1.2915) = -0.00475017278...$ M1

Sign change (and as $f(x)$ is continuous) therefore a root α is such that

$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp) A1 2

Note

M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x).
A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.

[9]

2.	(a)	$x(2x^2 - 1) = 4$	M1	
		$2x^2 - 1 = \frac{4}{x}$		
		$2x^2 = \frac{4+x}{x}$	M1	
		$x^2 = \frac{4+x}{2x}$		
		$x = \sqrt{\frac{2}{x} + \frac{1}{2}}$ **AG**	A1	3
		Alternative 1:		
		$2x^2 - 1 - \frac{4}{x} = 0$	M1	
		$2x^2 = 1 + \frac{4}{x}$		
		$x^2 = \frac{1}{2} + \frac{4}{2x}$	M1	
		$x\sqrt{\frac{1}{2} + \frac{2}{x}}$ **AG**	A1	
		Alternative 2:		
		$x^2 = \frac{2}{x} + \frac{1}{2}$	M1	
		$2x^3 = 4 + x$	M1	
		$2x^2 - x - 4 = 0$	A1	
	(b)	1.41, 1.39, 1.39 (1.40765, 1.38593, 1.393941)	B1,B1,B1	3
	(c)	$f(1.3915) = -3 \times 10^{-3}$	M1	
		$f(1.3925) = 7 \times 10^{-3}$	A1	
		change in sign means root between 1.3915 & 1.3925 $\therefore$ 1.392 to 3 dp	B1	3

[9]

3.	(a)	$f'(x) = 3e^x - \frac{1}{2x}$	M1A1A1	3
		M1: any evidence to suggest that tried to differentiate		

(b)	$3e^{\alpha} - \dfrac{1}{2\alpha} = 0$	M1	
	Equating f′ (x) to zero		
	$\Rightarrow 6\alpha\, e^{\alpha} = 1 \Rightarrow \alpha = \dfrac{1}{6}e^{-\alpha}$ AG	A1(cso)	2
(c)	$x_1 = 0.0613\ldots$, $x_2 = 0.1568\ldots$, $x_3 = 0.1425\ldots$, $x_4 = 0.1445\ldots$	M1A1	2
	M1 at least x_1 correct, A1 all correct to 4 d.p.		
(d)	Using f′(x) {$= 3\,e^x - \dfrac{1}{2x}$} with suitable interval	M1	
	[e.g. f(0.14425) = – 0.007, f(0.14435) = +0.002(1)] Both correct with concluding statement.	A1	2

[9]

4.	(a)	$f'(x) = -\dfrac{1}{2x^2}; + \dfrac{1}{x}$	M1A1;A1	
		M1 for evidence of differentiation. Final A – no extras		
		$f'(x) = 0 \Rightarrow \dfrac{-1+2x}{2x^2} = 0; \Rightarrow x = 0.5$	M1A1 (*) cso	5
		(or subst x = 0.5)		
	(b)	$y = 1 - 1 + \ln\left(\dfrac{1}{4}\right); = -2\ln 2$	M1; A1	2
		Sust 0.5 or their value for x in		
	(c)	f(4.905) = < 0 (–0.000955), f(4.915) = > 0 (+ 0.000874)	M1	
		evaluate		
		Change of sign indicates root between and correct values to 1 sf)	A1	2
	(d)	$\dfrac{1}{2x} - 1 + \ln\left(\dfrac{x}{2}\right) = 0;\ \Rightarrow 1 - \dfrac{1}{2x} = \ln\left(\dfrac{x}{2}\right)$		
		$\Rightarrow \dfrac{x}{2} = e^{\left(1-\frac{1}{2x}\right)}; \Rightarrow x = 2e^{\left(1-\frac{1}{2x}\right)}$ (*) (c.s.o.)	M1; A1	2
		M1 for use of e to the power on both sides		
	(e)	$x_1 = 4.9192$	B1	

		$x_2 = 4.9111$, $x_3 = 4.9103$,	B1	2
		both, only lose one if not 4dp		
				[13]
5.	(a)	f(1) = –2, f(2) = 5 ½	M1	
		Change of sign (and continuity) $\Rightarrow$ root $\in (1, 2)$	A1	2
	(b)	$x_1 = 1.38672\ldots$, $x_2 = 1.39609$ awrt 4dp	B1, B1	
		$x_3 = 1.39527\ldots$, $x_4 = 1.39534\ldots$ same to 3dp	M1	
		Root is 1.395 (to 3dp) cao	A1	4
	(c)	Choosing a suitable interval, (1.3945, 1.3955) or tighter.	M1	
		f(1.3945) ≈ – 0.005, f(1.3955) ≈ + 0.001		
		Change of sign (and continuity) $\Rightarrow$ root $\in$ (1.3945, 1.3955)		
		$\Rightarrow$ root is 1.395 correct to 3dp	A1	2
				[8]
6.	(a)	Attempting to reach at least the stage $x^2(x + 1) = 4x + 1$	M1	
		Conclusion (no errors seen) $x = \sqrt{\dfrac{4x+1}{x+1}}$ (*)	A1	2
		[Reverse process: need to square and clear fractions for M1]		
	(b)	$x_2 = \sqrt{\dfrac{4+1}{1+1}} =$ **1.58**…	M1	
		$\boldsymbol{x_3 = 1.68}$, $\boldsymbol{x_4 = 1.70}$	A1A1	3
		[Max. deduction of 1 for more than 2 d.p.]		
	(c)	Suitable interval; e.g. [1.695, 1.705] (or "tighter")	M1	
		f(1.695) = –0.037…, f(1.705) = + 0.0435…	Dep. M1	
		Change of sign, no errors seen, so root = 1.70 (correct to 2 d.p.)	A1	3
	(d)	$x = -1$, "division by zero not possible", or equivalent	B1,B1	2
		or **any number in interval –1 < *x* < –¼,** "square root of neg. no."		
				[10]
7.	(a)	$f^{-1}(x) = 0$ for maximum (or stationary point or turning point)	B1	
		$f^1(1.48) = e^{1.48} - 2 \times 1.48^2 = 0.0121\ldots$	M1	
		$f^1(1.49) =$ $= -0.0031\ldots$		
		change of sign $\therefore$ root / maximum in range	A1	3
		M1 May be $\Rightarrow$ if maximum mentioned at A1		

M1 One value correct to 1 S.F.
A1 Both correct and comment

(b)	$y = e^x - \frac{2}{3}x^3\ (+c)$	M1 A1	
	at (0, 5) $\quad 5 = e^0 - 0 + c$	M1	
	$\underline{c = 4}\left(y = e^x - \frac{2}{3}x^3 + 4\right)\ (c = 4)$	M1	4

M1 Some correct $\int$

A1 $e^x - \frac{2}{3}x^3$

M1 Attempt to use (0,5)
No + c is M0

(c)	Area = $\int_0^2 \left(e^x - \frac{2}{3}x^3 + 4\right)\mathrm{d}x$	M1	
	$= \left[e^x - \frac{2}{12}x^4 + 4x\right]_0^2$	A1ft	
	$= \left(e^2 - \frac{16}{6} + 8\right) - (e^{\ddot{o}} - 0 + 0)$	M1	
	$= \underline{e^2 + 4\frac{1}{3}}$ or $\underline{e^2 + \frac{13}{3}}$	A1 cao	4

M1 Some correct $\int$ *<u>other</u> than* $e^x \to e^x$.

A1 ft [ ] ft their $c(\neq 0)$.
M1 Attempt both limits

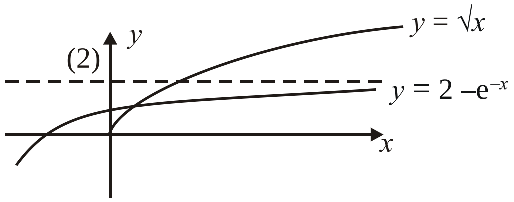
[11]

8.	(a)	$f'(x) = \frac{3}{x} - \frac{1}{x^2}$	M1 A1	
		$\frac{3}{x} - \frac{1}{x^2} = 0 \Rightarrow 3x^2 - x = 0 \Rightarrow x = \frac{1}{3}$	M1 A1	4
	(b)	$y = 3\ln\left(\frac{1}{3}\right) + \frac{1}{\left(\frac{1}{3}\right)} = 3 - 3\ln 3\ (k = 3)$	M1 A1	2

(c)	$x = 1 \Rightarrow y = 1$		B1	
	$f'(1) = 2 \Rightarrow m = -\dfrac{1}{2}$		M1	
	$y - 1 = -\dfrac{1}{2}(x - 1) \quad \left(y = -\dfrac{x}{2} + \dfrac{3}{2}\right)$		M1 A1	4
(d)	(i)	$-\dfrac{x}{2} + \dfrac{3}{2} = 3\ln x + \dfrac{1}{x}$	M1	
		leading to $6 \ln x + x + \dfrac{2}{3} - 3 = 0$ (*)	A1	
		cso		
	(ii)	g(0.13) = 0.273… g(0.14) = –0.370…	M1	
		Both, accept one d.p.		
		Sign change (and continuity) $\Rightarrow$ root $\in$ (0.13, 0.14)	A1	4

[14]

9. (a)



	[Ignore graph for y < 0, x < 0]		
	$y = \sqrt{x}$: starting (0,0)	B1	
	$y = 2 - e^{-x}$: shape & int. on + y-axis	B1	
	correct relative positions	B1	3
	[1 int^n $\sqrt{x}$ on top for $x \to \infty$]		
(b)	Where curves meet is solution to f(x) = 0; <u>only one intersection</u>	B1	1
(c)	f(3) = –0.218… f(4) = 0.018…	M1	
	one correct value to 1 sf		
	change of sign $\therefore$ root in interval	M1	2
	both correct (1 sf) + comment		

(d)	$x_0 = 4$	$x_1 = (2 - e^{-4})^2$	$= 3.92707\ldots$	M1	
		expression or x_1 to 3 dp			
		$x_2 =$	$3.92158\ldots$	A1	
		x_1, x_2 to ≥ 4 dp			
		$x_3 =$	$3.92115\ldots$		
		carry on to x_4			
		$x_4 =$	$3.92111(9)\ldots$	M1	
		to ≥ 3 dp			
	Approx. solution = 3.921 (3 dp)			A1 cao	4

[10]

1. All four parts of this question were well answered by the overwhelming majority of candidates who demonstrated their confidence with the topic of iteration with around 65% of candidates gaining at least 8 of the 9 marks available.

Some candidates in parts (a), (c) and (d) worked in degrees even though it was stated in the question that x was measured in radians.

In part (a), the majority of candidates evaluated both f(1.2) and f(1.3), although a very small number choose instead to evaluate both f(1.15)and f(1.35) A few candidates failed to conclude “sign change, hence root” as minimal evidence for the accuracy mark.

Most candidates found the proof relatively straightforward in part (b). A small number of candidates lost the accuracy mark by failing to explicitly write 4cosc$x$ – 4x + 1 as equal to 0 as part of their proof.

Part (c) was almost universally answered correctly, although a few candidates incorrectly gave x_1 as 1.3037 or x_3 as 1.2918.

The majority of candidates who attempted part (d) choose an appropriate interval for x and evaluated f(x) at both ends of that interval. The majority of these candidates chose the interval (1.2905, 1.2915) although incorrect intervals, such as (1.290, 1.292)were seen. There were a few candidates who chose the interval (1.2905, 1.2914). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 1.291 to 3 decimal places or α = 1.291 or even QED.

A minority of candidates who attempted part (d) by using a repeated iteration technique received no credit because the question required the candidate to consider a change of sign of f(x).

2. **Pure Mathematics P2**

A few candidates made no attempt to link the original function with the iteration formula. For those who did attempt this part, the most popular route was to start with f(x) = 0 and attempt to rearrange it. There were a few algebraic slips, and some candidates who could not see how to achieve an x^2 term, but most were successful. The few candidates who worked in the reverse direction, working from the iteration formula towards f(x) were all successful.

Many candidates were successful in applying the iteration formula, and usually noticed the instruction to give x_1, x_2 and x_3 to 2 decimal places. A few candidates appeared to think that this instruction applied only to x_3. There were some candidates having difficulty in the correct use of their calculators; a common set of false answers resulted from the alternative formula $\sqrt{\frac{2}{x}} + \frac{1}{2}$, despite the original formula being quoted correctly.

The wording of the final part of this question clearly indicates the need for a method involving an interval, yet many candidates simply continued for several more applications of the iteration formula. This approach gained no credit. Some candidates continue to have difficulty in identifying the correct values to use for the endpoints of their interval. (1.391, 1.393) was quite a common error, as was candidates attempting to add and subtract 5 in the wrong decimal place. There were a good number of totally correct answers.

Core Mathematics C3

This question was well answered and many candidates gained full marks. The great majority were able to provide the appropriate sequence of steps to demonstrate the result in part (a) and part (b) was nearly always completely correct with the answers given to the accuracy requested. The method tested in part(c) was clearly known to more candidates than had been the case in some previous examinations but there is still a minority of candidates who think that repeating the iteration gives a satisfactory proof. Here the question specified that an interval should be used and further iterations could gain no marks. The majority chose the appropriate interval (1.3915, 1.3925) although incorrect intervals, such as (1.391, 1.393) were seen. There were candidates who chose the interval (1.3915, 1,3924). This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the result and this was accepted for full marks. To gain the last mark, candidates are expected to give a reason for their conclusion, e.g. there is a sign change, and give a suitable conclusion such as that the root is 1.392 to 3 decimal places, the traditional, QED or, the modern, W.

3. Pure Mathematics P2

Most candidates were able to complete some, if not all parts of this question.

(a) Most candidates scored well, with an inability to differentiate log x being the most common cause of error. This part of the question was subject to a common misread of the function as $f(x) = 3e^x - \frac{1}{2}\ln(x-2)$. We often see candidates working without correct used of brackets, and here they saw brackets that were not present.

(b) have been straight forward but was made difficult by several candidates. Many candidates managed to produce the correct answer to (b) from a completely wrong answer to (a)!

(c) This was well done - most candidates obviously know how to use the ANS button on their calculators. Many candidates who went wrong had either made rounding errors, or they had assumed that $e^{-1} = 1$ when making their first substitution.

(d) This part was found more difficult. Candidates who did use an appropriate interval did not always give the values of the derivative correctly (for example, we assumed that an answer of 2.06… was a misread of a value in standard form). Some candidates did all the working correctly but did not draw any conclusion from it. Many chose an interval that did not include the root or that was too wide. A significant number ignored the wording of the question and continued the iteration process.

Core Mathematics C3

(a) Most candidates were confident differentiating both e^x and ln x. Some candidates misread f(x) and had ln(x – 2) instead of lnx -2. A number did not simplify their fraction to 1/(2x) but they were not penalised for this.

(b) They usually completed the rearrangement in part b) successfully although often needing several stages before reaching the answer on the paper. Some failed to replace the x with an alpha and a number had problems with their algebraic fractions.

(c) Candidates are very competent at obtaining values using an iteration formulae but some are not precise about the required number of decimal places.

(d) There was general familiarity with the change of sign method for determining a root but not always sufficient decimal places, and some intervals were too wide. Conclusions were sometimes missing. Some answers used f instead of f', and others used an iterative approach despite the instruction in the question.

4. In part (a) many candidates wrote $1/2x$ as $2x^{-1}$ and differentiated to get $2x^{-2}$. Many candidates then converted $-2x^{-2}$ to $-1/2x^{2}$. Much fudging then went on to arrive at $x = ½$, particularly since a large number had *d(ln x/2)/dx = 2/x*. A fair number tried to show f(1/2) = 0. Part (b) Most candidates substituted in x = ½ correctly but a few did not know what to do with $\ln\frac{1}{4}$.

Most candidates knew what to do in part (c), though a few candidates did not actually evaluate f(4.905) or f(4.915) or did so incorrectly. The phrase "change of sign" was often not mentioned, but replaced with long convoluted statements. Part (d) was well answered but some candidates did miss lines out going from $\ln\frac{x}{2} = 1 - \frac{1}{2x}$ to $x = 2e^{1-\frac{1}{2x}}$

In part (e) the majority of candidates did well with some students though some lost marks because they could not use their calculator correctly or did not give their answers to the required number of decimal places.

5. Part (a) was well answered but in part (b) many did not work to an appropriate accuracy. In order to give a final answer to 3 decimal places, it is necessary to produce intermediate working to 4 decimal places. Many produced an incorrect iteration which arose from the incorrect use calculators. Calculators differ, but most, when given an instruction (2 + 1/x)^1/3, calculate $\frac{\left(2+\frac{1}{x}\right)^1}{3}$ not $\left(2+\frac{1}{x}\right)^{\frac{1}{3}}$. For the second of these expressions, the instruction (2 + 1/x)^(1/3) is needed. A correct solution to part (c) requires the use of the interval (1.3945, 1.3955) or of an appropriate smaller interval. Less than 40% of the candidates recognised this.

6. Most candidates were able to gain marks, particularly in parts (a) and (b), although some lost accuracy marks. In part (c) many candidates did not read, or interpret, the question carefully enough, and the most common offerings were just to produce another three iterations, or to look for a sign change in an interval which was too wide. Only the better candidates gained full marks here. Answers to the last part were variable. Often a correct value with a wrong reason was seen, and "$x_1 = -¼$, because $x_2 = 0$, which is not valid" was quite common.

7. Most candidates were familiar with the type of question in part (a) but a few still failed to evaluate the derivative at 1.48 and 1.49.

Simply stating that $f'(1.48) > 0$ and $f'(1.49) < 0$ is not sufficient. Some candidates failed to appreciate the answer their calculator gave them was in standard form and –3.1 instead of –0.0031 was a common mistake. In part (b) most integrated successfully but some forgot to include the constant of integration and were not then able to use the point (0, 5) properly. There were still a few who substituted $\frac{\mathrm{d}y}{\mathrm{d}x}$ into $y - y_1 = m(x - x_1)$.

The technique required in part (c) was well known but many candidates failed to heed the instruction to give the exact area.

8. Part (a) was usually well done by those who knew what a stationary value was although minor algebraic errors often spoilt essentially correct solutions. A few candidates either had no idea what a stationary value was, and began the question at part (c), or thought that they needed the second, rather than the first, derivative equal to zero. Part (b) proved the hardest part of this question and many were not able to use a correct law of logarithms to find k. Part (c) was very well done but part d(i) produced some rather uncertain work. Many candidates did not seem to realise what was expected. Nearly all candidates knew the technique required in part (d)(ii), although a few substituted 0.13 and 0.14 into f(x). The examiners do require a reason and a conclusion at the end of such a question.

9. The sketches in part (a) proved quite demanding for some candidates. The sketch of $y = \sqrt{x}$ usually passed through the origin but the curvature was often incorrect and a sketch that resembled $y = e^{-x}$ was often given for $y = 2 - e^{-x}$. Part (b) was usually answered correctly but a small number of candidates thought that this question was more to do with the number of intersections with the axes rather than between the two curves. Parts (c) and (d) are now well rehearsed by candidates on this paper and fully correct solutions were the norm. A few failed to comment on the change of sign in part (c) and there were some errors in accuracy in part (d). A very small number of weaker candidates in part (d) simply substituted 1, 2, 3 and 4 for x.